

Selections from Frege's Correspondence

Gottlob Frege (1848–1925) and Bertrand Russell (1872–1970) are generally considered to be among the founders of analytic philosophy. Below is included some of the first, seminal correspondence between them. Translated by Hans Kaal (G. Frege, *Philosophical and Mathematical Correspondence* [Blackwell: Oxford, 1980], 130ff) with tiny adaptations.

Russell to Frege 16th June 1902

I have known your *Basic Laws of Arithmetic* for a year and a half, but only now have I been able to find the time for the thorough study I intend to devote to your writings. I find myself in full accord with you on all main points, especially in your rejection of any psychological element in logic and in the value you attach to a conceptual notation for the foundations of mathematics and of formal logic, which, incidentally, can hardly be distinguished. On many questions of detail, I find discussions, distinctions and definitions in your writings for which one looks in vain in other logicians. On functions in particular (sect. 9 of your *Conceptual Notation*) I have been led independently to the same views even in detail. I have encountered a difficulty only on one point. You assert (p. 17) that a function could also constitute the indefinite element. This is what I used to believe, but this view now seems dubious because of the following contradiction: Let w be the predicate of being a predicate which cannot be predicated of itself. Can w be predicated of itself? From either answer follows its contradictory. We must therefore conclude that w is not a predicate. Likewise, there is no class (as a whole) of those classes which, as wholes, are not members of themselves. From this I conclude that under certain circumstances a definable set does not form a whole.

I am in the process of completing a book on the principles of mathematics, and I should like to discuss your work in it in great detail. I already have your books, or I shall buy them soon; but I should be very grateful to you if you could send me offprints of your articles in various journals. But if this should not be possible, I shall get them from a library.

On the fundamental questions where symbols fail, the exact treatment of logic has remained very backward; I find that yours is the best treatment I know in our time; and this is why I have allowed myself to express my deep respect for you. It is very much to be regretted that you did not get around to publishing the second volume of your *Basic Laws*; but I hope that this will still be done.

Yours sincerely,
Bertrand Russell

The above contradiction can be expressed in Peano's notation as follows:

$$w = \text{cls } \cap x \ni (x \sim \in x) . \supset : w \in w . = .w \sim \in w.^1$$

I have written about this to Peano, but he still owes me a reply.

¹ Translation: if w is the class of x such that x is not an element of x , then w is an element of w if and only if w is not an element of w .

Frege to Russell 22nd June 1902

Dear Colleague,

Many thanks for your interesting letter of 16 June. I am glad that you agree with me in many things and that you agree with me in many things and that you intend to discuss my work in detail. In accordance with your wishes I am sending you the following offprints:

- (1) 'Critical Elucidation etc.'
- (2) 'On the Notation of Mr Peano etc.'
- (3) 'On Concept and Object'
- (4) 'On Sense and Reference'
- (5) 'On Formal Theories of Arithmetic'

I have received an empty envelope addressed in what seems to be your handwriting. I suspect that you had the intention of sending me something, but that it got lost by accident. If this is the case, I thank you for your good intention. I am enclosing the front of the envelope.

When I now reread my *Conceptual Notation*, I find that I have changed my view on some points, as you will see if you compare it with my *Basic Laws of Arithmetic*. Please cross out the paragraph on p. 7 of my *Conceptual Notation* beginning with 'We can just as easily' because it contains a mistake which, incidentally, did not have any undesirable consequences for the rest of the contents of my little book.

Your discovery of the contradiction has surprised me beyond words and, I should almost like to say, left me thunderstruck, because it has rocked the ground on which I meant to build arithmetic. It seems accordingly that the transformation of the generality of an identity into an identity of ranges of values (sect. 9 of my *Basic Laws*) is not always permissible, that my law V (sect. 20, p. 36) is false, and that my explanation in sect. 31 do not suffice to secure a meaning for my combinations of signs in all cases. I must give some further thought to the matter. It is all the more serious as the collapse of my law V seem to undermine not only the foundations of my arithmetic but the only possible foundations of arithmetic as such. And yet, I should think, it must be possible to set up conditions for the transformation of the generality of an identity into an identity of ranges of values so as to retain the essentials of my proofs. Your discovery is at any rate a very remarkable one, and it may perhaps lead to a great advance in logic, undesirable as it may seem at first sight.

Incidentally, the expression 'A predicate is predicated of itself' does not seem exact to me. A predicate is as a rule a first-level function which requires an object as argument and which cannot therefore have itself as argument (subject). Therefore I would rather say: 'A concept is predicated of its own extension'. If the function $\Phi(\xi)$ is a concept, I designate its extension (or the pertinent class) by ' $\hat{\epsilon}\Phi(\xi)\epsilon$ ' (though I now have some doubts about the justification for this). ' $\Phi(\hat{\epsilon}\Phi(\epsilon))$ ' or ' $\hat{\epsilon}\Phi(\epsilon) \cap \hat{\epsilon}\Phi(\epsilon)$ ' is then the predication of the concept $\Phi(\xi)$ of its own extension.

The second volume of my *Basic Laws* is to appear shortly. I shall have to give it an appendix where I will do justice to your discovery. If only I could find the right way of looking at it!

Yours sincerely,
G. Frege

Below is included a letter written by Frege to Philip Jourdain (1879–1919), editor of the *Monist*, in which Frege responds to Jourdain's questions (one of which was about whether *Sinn* was a psychological feature of a name) and offers a useful account of how he himself understood his account of 'sense' (*Sinn*).

Frege to Jourdain [undated earlier draft of a letter sent on the 28th January 1914]

I am very glad to give you permission to translate parts of my *Basic Laws* for the *Monist*. From your letter it seems to me Mr Wittgenstein is again in Cambridge. I had lengthy conversations with him before Christmas, and I wanted to write him a letter about them in order to carry on the thread, but I did not know where he was. Unfortunately I do not understand the English language well enough to be able to say definitely that Russell's theory (*Principia Mathematica* I, 54ff) agrees with my theory of functions of the first, second, etc. levels. It does seem so. But I do not understand all of it. It is not quite clear to me what Russell intends with his designation $\Phi!x$. I never know for sure whether he is speaking of a sign or of its content. Does 'function' mean a sign? I already wrote to you once why I wanted to see the expression 'variable' banned. One never knows exactly whether it is supposed to be a sign or the content of a sign.

[...] With regard to your second question I want to say the following. Judging (or recognizing as true) is certainly an inner mental process; but that something is true is independent of the recognizing subject; it is objective. If I assert something as true I do not want to talk about myself, about a process in my mind. And in order to understand it one does not need to know who asserted it. Whoever understands a proposition (*Satz*) uttered with assertoric force adds to it his recognition of the truth. If a proposition uttered with assertoric force expresses a false thought, then it is logically useless and cannot strictly speaking be understood. A proposition uttered without assertoric force can be logically useful even though it expresses a false thought, e.g., as part (antecedent) of another proposition. What is to serve as the premise of an inference must be true. Accordingly, in presenting an inference, one must utter the premises with assertoric force, for the truth of the premises is essential to the correctness of the inference. If in representing an inference in my conceptual notation one were to leave out the judgement strokes before the premised propositions, something essential would be missing. And it is good if this essential thing is visibly embodied in a sign and not just added to it in the act of understanding according to a tacit convention; for a convention according to which something has to be added in that act of understanding under certain circumstances is easily forgotten even if it was once stated explicitly. And so it happens that something essential is completely overlooked because it has not found an embodiment. But what is essential to an inference must be counted as part of logic.

As far as your third question is concerned, I do not believe that we can dispense with the sense of a name in logic; for a proposition must have a sense if it is to be useful. But a proposition consist of parts which must somehow contribute to the expression of the sense of the proposition, so they themselves must somehow have a sense. Take the proposition 'Etna is higher than Vesuvius'. This contains the name 'Etna', which occurs also in other propositions, e.g. in the proposition 'Etna is in Sicily'. The possibility of our understanding propositions which we have never heard before rests evidently on this, that we construct the sense of a proposition out of parts that correspond to the words. If we find the same word in two propositions, e.g., 'Etna', then we also recognize something common to the

corresponding thoughts, something corresponding to this word. Without this, language in the proper sense would be impossible. We could indeed adopt the convention that certain signs were to express certain thoughts, like railway signals ('The track is clear'); but in this way we would always be restricted to a very narrow area, and we could not form a completely new proposition, one which would be understood by another person even though no special convention had been adopted beforehand for this case. Now that part of the thought which corresponds to the name 'Etna' cannot be Mount Etna itself; it cannot be the meaning of this name. For each individual piece of frozen, solidified lava which is part of Mount Etna would then also be part of the thought that Etna is higher than Vesuvius. But it seems to me absurd that pieces of lava, even pieces of which I had no knowledge, should be parts of my thought. Thus both things seem to me necessary: (1) the reference (*Bedeutung*) of a name, which is that about which something is being said, and (2) the sense (*Sinn*) of the name, which is part of the thought. Without a referent (*Bedeutung*), we could indeed have a thought, but only a mythological or literary thought, not a thought that could further scientific knowledge. Without a sense, we would have no thought, and hence also nothing that we could recognize as true.

To this can be added the following. Let us suppose an explorer travelling in an unexplored country sees a high snow-capped mountain on the northern horizon. By making inquiries among the natives he learns that its name is 'Aphla'. By sight it from different points he determines its position as exactly as possible, enters it in a map, and writes in a diary: 'Aphla is at least 5000 metres high'. Another explorer sees a snow-capped mountain on the southern horizon and learns that it is called Ateb. He enters it in his map under this name. Later comparison shows that both explorers saw the same mountain. Now the content of the proposition 'Ateb is Aphla' is far from being a mere consequence of the principle of identity, but contains a valuable piece of geographical knowledge. What is stated in the proposition 'Ateb is Aphla' is certainly not the same thing as the content of the proposition 'Ateb is Ateb'. Now if what corresponded to the name 'Aphla' as part of the thought was the reference (*bedeutung*) of the name and hence the mountain itself, then this would be the same in both thoughts. The thought expressed in the proposition 'Ateb is Aphla' would have to coincide with the one in 'Ateb is Ateb', which is far from being the case. What corresponds to the name 'Ateb' as part of the thought must therefore be different from what corresponds to the name 'Aphla' as part of the thought. This cannot therefore be the meaning which is the same for both names, but must be something which is different in the two cases, and I say accordingly that the sense of the name 'Ateb' is different from the sense of the name 'Aphla'. Accordingly, the sense of the proposition 'Ateb is at least 50000 metres high' is also different from the sense of the proposition 'Aphla is at least 5000 metres high'. Someone who takes the latter to be true need not therefore take the former to be true. An object can be determined in different ways, and every one of these ways of determining it can give rise to a special name, and these different names then have different senses; for it is not self-evident that it is the same object which is being determined in different ways. We find this in astronomy in the case of planetoids and comets. Now if the sense of a name were something subjective, then the sense of the proposition in which the name occurs, and hence the thought, would also be something subjective, and the thought one person connects with this proposition would be different from the thought another connects with it; a common store of thoughts, a common science would be impossible. It would be impossible for something one person said to contradict what another said, because the two would not express the same thought at all, but each his own.

For these reasons I believe that the sense of a name is not something subjective², that it does not therefore belong to psychology, and that it is indispensable.

² Note that here, crossed out, is 'in one's mental life'.